Supplementary material (Appendix B):

"Spillover feedback loops and strategic complements in R&D"

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Bertrand competition and optimal R&D

We examine the effects of spillovers in a Bertrand setting with differentiated products. The main difference in firms' interactions in the Bertrand and Cournot settings is that for Bertrand rivals, product market competition gives rise only to detrimental effects on the innovator's profit. This happens because cost-reducing innovation allows the R&D-taking firm to set a lower price. Competing for market share, a rival responds by also reducing its price, leading to lower profits for the innovator. Thus, due to the price war in the product market, outgoing spillovers that strengthen a rival's strategic position should harm the R&D-taking firm. However, in the RF model, a positive relationship between outgoing spillovers and optimal R&D can also hold for Bertrand rivals. To perform this analysis, we employ a demand system obtained by inverting the inverse demands as in Singh & Vives (1984), and Vives (1984).

Suppose that there is some degree of differentiation between rivals' goods and firm i's Cournot demand is $p_i = a - q_i - bq_j$, where $b \in (0,1)$. To analyze a product market that involves Bertrand competition, we rewrite each firm i's Cournot demand with respect to prices and obtain $q_i = \frac{1}{1+b}\left[a - \frac{1}{1-b}\left(p_i - bp_j\right)\right]$. Then, we recursively solve the game. In period 2, we derive firm i's equilibrium price, $p^i_{2,R} = \frac{1}{2-b}\left[a\left(1-b\right) + \frac{1}{2+b}\left(2c_i + bc_j\right)\right]$, and its net profit, $\pi^i_{2,B} = \left(1-b^2\right)\left(q^i_{2,B}\right)^2 - g\left(x^i_{2,B}\right)$, where

$$q^i_{2,B} = \frac{\alpha - \overline{c}}{\left(2-b\right)\left(1+b\right)} + \gamma_{i,B}\left(x^i_{1,B} + x^i_{2,B}\right) - \frac{b - \beta_i\left(2-b^2\right)}{\left(4-b^2\right)\left(1-b^2\right)\left(1-\beta_i\beta_j\right)}\left(x^j_{1,B} + x^j_{2,B}\right), \qquad (1)$$

and $\gamma_{i,B} \equiv \frac{2-b\left(b+\beta_j\right)}{\left(4-b^2\right)\left(1-b^2\right)\left(1-\beta_i\beta_j\right)}$. The subscript B indicates the equilibrium R&D decisions when firms compete à la Bertrand in the RF model. Thus, a firm i's R&D decision is a strategic

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complement when $\frac{b}{2-b^2} < \beta_i$, and a strategic substitute otherwise. The following assumptions guarantee the existence of a unique interior equilibrium:

$(B.1)$ $\delta_{i,B} \equiv 4\left(1-b^2\right)\gamma_{i,B}^2 - g'\left(x_{2,B}^i\right) < 0, \forall i, j$

$(B.2)$ $\Delta_B \equiv \delta_{i,B}\delta_{j,B} + 4\left(4-b^2\right)^2 \gamma_{i,B}\gamma_{j,B}\left[\beta_i\left(2-b^2\right)-b\right]\left[\beta_j\left(2-b^2\right)-b\right] > 0$

Firm i's first-order condition gives

$$2\left(1-b^2\right)\gamma_{i,B} q_{2,B}^i - g'\left(x_{2,B}^i\right) = 0,$$

where $q_{2,B}^i$ is given in (1). Totally differentiating firm i's optimal R&D incentives with β_j yields

$$\frac{2\left[\beta_i\left(2-b^2\right)-b\right]}{(2+b)\left(1-\beta_i\beta_j\right)^2\left(4-b^2\right)}\left[q_{2,B}^i + \gamma_{i,B}\left(\sum_{t=1}^{2} x_{t,B}^{i*} + \beta_i \sum_{t=1}^{2} x_{t,B}^{j*}\right) + \frac{2-b\left(b+\beta_j\right)}{(2+b)(1-b)}\frac{dx_{2,B}^{j*}}{d\beta_j}\right] + \delta_{i,B}\frac{dx_{2,B}^{i*}}{d\beta_j} = 0. \tag{2}$$

Totally differentiating firm j's optimal R&D incentives with β_j yields

$$\frac{2\left(1-b^2\right)\gamma_{j,B}}{1-\beta_i\beta_j}\left[\beta_i q_{2,B}^j + \beta_i \sum_{t=1}^{2} x_{t,B}^{j*} + \sum_{t=1}^{2} x_{t,B}^{i*} + \frac{\beta_j\left(2-b^2\right)-b}{\left(4-b^2\right)\left(1-b^2\right)}\frac{dx_{2,B}^{i*}}{d\beta_j}\right] + \delta_{j,B}\frac{dx_{2,B}^{j*}}{d\beta_j} = 0. \tag{3}$$

By equations (2) and (3), we get

$$\frac{dx_{2,R}^{i*}}{d\beta_j} = \frac{2\left[\beta_i\left(2-b^2\right)-b\right]\Psi_B}{\left(1-\beta_i\beta_j\right)^2(2+b)\left(4-b^2\right)\Delta_B},$$

where

$$\begin{aligned}\Psi_B \equiv\ & \frac{2(1+b)\left[2-b\left(b+\beta_j\right)\right]}{(2+b)\left(1-\beta_i\beta_j\right)^2}\gamma_{j,B}\left(\beta_i q_{2,B}^j + \beta_i \sum_{t=1}^{2} x_{t,B}^{j*} + \sum_{t=1}^{2} x_{t,B}^{i*}\right) \\ & -\delta_{j,B}\left[q_{2,B}^i + \gamma_{i,B}\left(\sum_{t=1}^{2} x_{t,B}^{i*} + \beta_i \sum_{t=1}^{2} x_{t,B}^{j*}\right)\right].\end{aligned}$$

Assumptions $(B.1)$ and $(B.2)$ guarantee that $\Delta_B > 0$ and $\delta_{j,B} < 0$. Hence, we have $\Psi_B > 0$, implying $\frac{dx_{2,B}^{i*}}{d\beta_j} > 0$, if and only if $\beta_i > \frac{b}{2-b^2}$ for all β_j. In the RF model with Bertrand competition among differentiated-products rivals, a firm conducts more R&D in equilibrium as outgoing spillovers increase if and only if its R&D best-response curve is upward sloping.

When β_i and β_j are high enough so that their positive effects on efficiency enhancement dominate their negative effects on the innovator's strategic position, a positive relationship between a firm's equilibrium R&D and outgoing spillovers can be realized in both Bertrand and Cournot settings: it does not depend on the mode of competition in the product market. However, in the R&D stage, the innovator's R&D decision needs to be a strategic complement.

References

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