Incentive contracts under product market competition and R&D spillovers[*]

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Abstract

This paper studies cost-reducing R&D incentives in a principal-agent model with product market competition. It argues that moral hazard does not necessarily decrease firms' profits in this setting. In highly competitive industries, firms are driven by business stealing incentives and exert such high levels of R&D that burn up their profits. In the presence of moral hazard, underprovision of R&D incentives due to risk-sharing can generate considerable cost-savings implying higher profits for both rivals. This result indicates firms' incentives to adopt a collusive-like behavior in the R&D market. We also examine the agents' contracts and the profits-risk relationship when cross-firm R&D spillovers occur.

Keywords: moral hazard, process innovation, Cournot competition, R&D spillovers, relative performance

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1 Introduction

In knowledge-based industries, firms' interactions and technical advance favor a decentralized organizational structure that involves separation between business units and research teams. The owners of firms appoint highly-skilled researchers or autonomous units to undertake cost-reducing R&D projects on their behalf. Thus, there is a division between ownership and control over R&D-outputs. In such markets, the issue of incentive provision deserves special attention when firms also compete in the product market. The principal-agent literature based on Holmström (1979) remains narrow in its focus on the effect of moral hazard and risk on firms' profits when there are strategic interactions among firms.

This paper examines whether the standard result in the literature that firms enjoy higher profits under full information applies in this setting. We argue that moral hazard does not necessarily decrease firms' profits. The conventional wisdom in models with moral hazard, originating from Holmström (1979) and Holmström & Milgrom (1987), is that the optimal contract balances an increase in risk with weaker incentives for effort due to risk-sharing. Thus, the owners of the firms are better-off under full information where no insurance is provided. We argue that the latter result need not hold if firms interact in the product market. We take into account the market environment and identify the conditions under which the profit-risk relationship turns out to be positive.

We consider a setting with two risk-neutral firms that first invest in cost-reducing R&D and then, interact in a differentiated-final product market. To conduct R&D, the owner of each firm (the principal) appoints a risk-averse researcher (the agent) whose effort is unobservable. The bargaining power is assigned to the principals allowing them to make take-it-or-leave-it offers to the agents and extract the entire rents of R&D activity. The incentive packages are derived in a linear principal-agent model (Holmström & Milgrom (1987)) and the payments are contingent on marginal cost reductions (Raith (2003)).[1,2] Each agent's R&D output depends on her own effort and a project-specific shock.

We derive the optimal R&D incentives and show that in highly competitive industries, firms are driven by business stealing incentives and exert such high levels of R&D that they burn up their profits. In the presence of moral hazard, risk-sharing mitigates such R&D incentives and firms' appetite for innovation. Lower effort is exerted implying cost savings for both rivals. We argue that there exists a regime in which cost savings are substantial so that firms' profits are higher under moral hazard. This occurs when the product market competition is intensive and the cost of R&D is relatively small. This paper delves into firms' incentives to adopt a collusive-like behavior in R&D and even utilize the intra-firm conflicts of interests. Separation of business and research units under moral hazard, prior to product market competition, can be used as a collusive device that mitigates firms' interactions in the proceeding stages. Firms become better-off as more insurance has to be

[1] Prendergast (1999) provides a review of the principal-agent literature.

[2] Cost-based schemes are consistent with real-world contracting practices. In Germany, for instance, inventors' compensation schemes based on the expected value of the R&D-outputs have been established by law (German Employees' Inventions Act passed in 1957).

provided to researchers.

We also examine cost-reducing R&D motives and the effect of risk on equilibrium profits when R&D spillovers occur. Each firm's cost reduction now also depends on the size of the spillovers; i.e. on the amount of (unpaid) appropriation of a rival's R&D (D'Aspremont & Jacquemin (1990), Kamien, Muller & Zang (1992), Qiu (1997), Amir, Amir & Jin (2000), among others). Due to technological interactions between agents, each principal now offers a relative performance evaluation scheme. The explicit comparison of R&D performances is the consequence of the efficient use of information conveyed by the individual R&D-outputs about each agent's effort. The existing literature uses such contracts when the market shocks that hit each agent's production are correlated (Holmström & Milgrom (1987)). In this model, there is no correlation between the random factors, however, spillovers necessitate the use of such schemes. In equilibrium, a negative weight is placed on a rival firm's performance, implying that an agent is penalized if the rival does better. Such contracts introduce 'competition' between agents and can effectively filter out spillovers from their compensation packages.

To study how profits change with risk in this context, we first discuss the effects of competition and spillovers on R&D incentives. In particular, the relative location of the firms in the product and technology space determines the 'nature' of strategic interactions in the R&D market. The analysis performs a decomposition of R&D incentives and focuses on the underlying effects that arise due to product market competition. These are the (positive) strategic effect due to business stealing and the (negative) spillover effect due to knowledge transmission. The latter effect is detrimental to the R&D-taking firm; i.e. spillovers enhance the efficiency of the rival making this firm tougher in the product market. If the strategic effect dominates the spillover effect, efforts are strategic substitutes.[3]

We show that in a regime where efforts are strategic substitutes, competition stimulates R&D in markets with highly elastic demand. This happens because a firm with cost-advantage can more easily extend its business at the expense of its rival.[4] Thus, each firm has stronger incentives to conduct R&D as demand becomes more elastic. In this regime, spillovers also foster R&D, if the cost of effort exertion is relatively small.[5] By investing in R&D, each principal wants to realize a slightly lower marginal cost from its competitor. As spillovers increase, rivals acquire more in R&D in order exactly to secure a cost advantage. Thus, both competition and spillovers induce firms to 'overinvest' in R&D. However, by doing so, R&D costs increase, without a commensurate increase in equilibrium profits. Differently, the presence of moral hazard on the part of the researchers leads to

[3]Bulow, Geanakoplos & Klemperer (1985) provide a taxonomy of strategic variables.

[4]Aghion, Bloom, Blundell, Griffith & Howitt (2005), Scherer & Ross (1990), among others, study empirically the effect of competition on incentives. Baggs & De Bettignies (2007), and Griffith (2001) examine the relationship between competition and agency cost.

[5]Levin (1988), among others, reports extensive spillovers mainly in bioengineering and microelectonics-based industries. Computer software, chemical compounds, genetic sequences are subject to spillovers due to disclosure of knowledge through publications or patents, researchers' mobility or even embodiment of knowledge in products (knowledge acquisition by reverse engineering). Bondt (1997) provides a review about the effect of knowledge spillovers on R&D investments.

underprovision of R&D incentives. A cost-saving choice for both rivals is to delegate R&D decisions ex-ante or even to appoint highly risk-averse agents in order to innovate less in R&D and thereby enjoy higher profits. Firms become better-off as the trade-off between effort provision and insurance is shifted towards the latter. We find that principals capitalize on such benefits only if the cost of incentivizing and insuring the researchers from the stochastic nature of their effort does not exceed a threshold. If the R&D activity is too costly, rivals' profits are higher under full information. This result sheds insight on the organizational structure firms may desire to adopt, given the cost of exerting effort and the market characteristics.

Gains from risk-sharing are also generated for firms that compete à la Bertrand in the product market. Firms enter into a price war. By being more efficient, they end up cutting prices, thereby diminishing their equilibrium profits. In this setting, investing less in R&D due to moral hazard can also increase the firms' profits. Thus, a positive profit-risk relationship can be realized in both Bertrand and Cournot settings: it does *not* depend on the mode of competition in the product market. For this to happen, agents' efforts must be strategic substitutes. If they are strategic complements, firms wish to undertake research under full information and effectively monitor the agents in order to exploit all opportunities from efficiency enhancement.

This analysis contributes to the existing literature on the theory of the firm that argues that considering a firm in isolation may be misleading. Strategic interactions play a key role in the firms' internal organization. This literature is based on Fershtman & Judd (1987) and Sklivas (1987). It focuses on "strategic delegation" and examines the effect of product market competition of the agents' compensation schemes and incentives.[6] From another perspective, Aggarwal & Samwick (1999) study how agents' incentives can influence the intensity of the strategic interactions between firms. These papers assume that agents perform in the product market and their compensation is contingent on firms' profits and sales. Effort is observable and there are no agency problems.[7] In this model, we assume that researchers' tasks are focused on cost reduction and their rewards are directly related to the output of their task. We use the standard principal-agent model in a competitive setting where the researchers' decisions cannot affect firms' strategic interactions.

The severity of the principal-agent problem when it is faced by product market competitors has been examined by Hart (1983), Scharfstein (1988), Hermalin (1992), Schmidt (1997), and Raith (2003), among others. Raith (2003) points out the difference between the risks firms face and the risks to which agents are exposed. He considers an endogenous number of firms that compete in prices along a Salop circle. He argues that incentives are positively related only to firm risk because changes in competition change the value of cost reductions and the variance of firms' profits in the

[6] Other works examine the effect of competition on incentives by considering changes in the number of competitors (Schmidt (1997)), the market size, the transportation cost or the cost of entry (Raith (2003)). Nickell (1996) and Vickers (1995) review the existing literature about the effect of competition on incentives and Vives (2008) provides a survey about the effect of competition on innovation. Milliou & Petrakis (2011) study the technology adoption incentives of market rivals.

[7] Hart (1983) and Piccolo, D'Amato & Martina (2008) also assume that the contracts are contingent on firms' profits.

same direction. In our model, we examine the risk faced by agents and argue that higher degrees of risk aversion and the riskiness of the performance measures decrease the R&D incentives but they can increase rivals' profits. More recently, Serfes (2008) derives a positive profits-risk relationship in an endogenous matching model with heterogeneous principals and agents.[8]

This paper can also be tied to the literature on firms' incentives to vertically integrate. If the R&D and production units are separate, contracts are used to govern their relationship and the moral hazard problem is present. This paper shows that in highly competitive markets where profits increase with risk, vertical separation is preferable. In contrast, in a regime where competition is soft and the profits-risk relationship becomes negative, firms have incentives to vertically integrate and the moral hazard problem disappears. Several recent papers explore firm boundaries and internal organization (i.e., Hart & Holmström (2010), Aghion, Dewatripont & Rey (2004), Alonso, Dessein & Matouschek (2008)). Aghion, Griffith & Howitt (2006) provide evidence of a U-shaped relationship between product market competition and vertical integration.

An additional contribution of this paper is the following. Holmström (1979) shows that the certainty equivalence of agent's utility can be written in the mean-variance form, if constant absolute risk averse preferences are considered, linear contracts are used and the random terms are normally distributed. The optimal effort only affects the first two moments of the distribution of wages and the agent's problem has a closed-form solution. We prove that this is also the case if the random terms follow a *truncated* normal distribution that is symmetric around the mean. Truncation is required in order to guarantee positive post-innovation marginal costs. This assumption is essential in all models based on Holmström (1979) that consider cost-reducing incentives under moral hazard.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 solves the rivalry game. It analyzes the underlying effects of risk on profits and interprets the results. In section 4, we study the incentive contracts and profit-risk relationships in the presence of spillovers. The cost-reducing motives of Bertrand rivals and the optimal contracts when there are two forms of correlation between the researchers' R&D outputs - i.e., due to spillovers and the correlation of the random terms - are also discussed. Section 5 concludes.

2 The model

The market features two risk-neutral and profit-seeking firms 1 and 2, indexed by i and j where $i \neq j$. Each firm is run by a principal whose task is first to invest in cost-reducing R&D and then, to make output decisions. To acquire R&D, each principal hires a risk-averse researcher, whose effort is unobservable and non-contractible. Thus, the principal's problem is to offer a contract based on contractible measures that is incentive compatible. The parties interact and play the three-stage

[8] Serfes (2005) assumes a continuum of principals and agents with uniform distributions and studies the relationship between risk and performance pay (incentives) in a principal-agent market. He finds the conditions under which the equilibrium relationship between risk and incentives is negative, positive, or non-monotonic.

game described in Figure 1.

Stage 1	→	Stage 2	→	Stage 3
Each principal, simultaneously with its rival, makes a take-it-or-leave-it offer to an agent.		If the agent accepts the offer, she chooses an effort level (x_i). Events beyond the agents' control occur, R&D-outputs are determined, and contracts are executed. If the agent rejects the offer, no innovation occurs.		Firms observe the R&D-outputs and compete à la Cournot in the product market. Profits are realized.

Figure 1. Timing of the game

2.1 Firm's profits

The market is populated by a continuum of identical consumers with mass equal to 1. Each firm i faces the linear demand $p_i = A - q_i - bq_j$ where p_i is firm i's price, $p_i : R_+^2 \to R_+$, and q_i is its output.[9] The parameter A denotes the market size, $A > 0$, and b captures the degree of product substitutability, $b \in [0, 1]$. When $b = 0$, firms have independent demands and behave as monopolists while, at the other extreme, when $b = 1$, they act as homogeneous-product duopolists. A higher b indicates tougher competition.

Each firm begins with an initial marginal cost $\bar{c}$, but takes advantage of cost-reducing R&D opportunities. If firm i acquires the R&D output z_i, it realizes the post-innovation marginal cost $c_i = \bar{c} - z_i$ where $A > \bar{c} > 0$. The R&D output, z_i, depends on the agent's effort, x_i, and a project-specific shock, ε_i, taking the form $z_i = x_i + \varepsilon_i$.[10] The random terms are drawn from a truncated normal distribution with zero mean and variance σ^2. They lie in $\Theta \equiv [-\theta, \theta]$, where $-\infty < -\theta < \theta < +\infty$, and are identically and independently distributed across agents.[11] Thus, firm i will commit to an R&D level $x_i \in X$ where $X \equiv [0, \bar{c} - \theta]$ and enjoy the R&D profit $\pi_i = \Pi_i - w_i$ where Π_i is the Cournot profit and w_i is the agent i's compensation.

2.2 Agents' compensation and preferences

Agent i has constant absolute risk-averse (CARA) preferences with utility function

$$U_i(w_i, x_i) = -e^{-r[w_i - \psi(x_i)]}, \tag{1}$$

where r is the Arrow-Pratt measure of risk aversion, $r > 0$, and $\psi(x_i)$ is the cost-of-effort function. This function is twice continuously differentiable and convex. Following Holmström (1979), the agent

[9] Following Singh & Vives (1984), the representative consumer's preferences are described by the standard quadratic utility function $V(q_i, q_j) = A(q_i + q_j) - \left[\frac{1}{2}\left(q_i^2 + q_j^2\right) + bq_iq_j\right]$. This function is separable and linear in the numeraire good. There are no income effects, and thus we can perform partial equilibrium analysis.

[10] Instead of process (cost-reducing) innovation, one could consider product innovation; i.e., quality improvement in existing products. Product innovation can be represented by an increase in consumers' willingness to pay captured by the parameter A. Firms' profit functions remain the same implying that the optimal choices and the comparative statics in our model apply in both settings (Vives (2008)).

[11] The value of θ is specified in subsection 3.2 where assumptions on the profit functions are made.

receives a linear contract that is contingent on her R&D output and generates a payment

$$w_i = \alpha_i + \beta_i z_i, \tag{2}$$

where α_i denotes the fixed salary component and β_i is a pay-for-performance parameter, $\beta_i \geq 0$. If the agent rejects the offer, she picks the outside option, which is normalized to zero.

3 Equilibrium and R&D incentives

We recursively solve the game and derive the subgame perfect Nash equilibrium. Firms make their decisions simultaneously and independently. We also analyze the effect of moral hazard on agents' effort and competitors' equilibrium profits.

3.1 Cournot competition

In stage 3, firms observe the realization of the marginal costs and compete in outputs. In particular, firm i maximizes $\Pi_i = [A - q_i - bq_j - c_i]\, q_i$ and produces

$$q_i^* = \frac{1}{2+b}\left[A - \bar{c} + \frac{2z_i - bz_j}{2-b}\right]. \tag{3}$$

Its Cournot profit is $\Pi_i^* = (q_i^*)^2$. Note that firms generically end up in an asymmetric equilibrium $\left(q_i^*, q_j^*\right)$ even if the R&D decisions taken in the previous stages were identical. This reflects that firms may experience asymmetric marginal costs depending on how lucky the researchers were during the R&D process; i.e. the realizations of ε_i and ε_j may differ.

3.2 Principals' problem and R&D rivalry

To acquire R&D, each principal, simultaneously with her rival, makes a contract offer to her agent that maximizes the expected profit and is compatible with agent's incentives to perform and to participate. Thus, the principal i's contract decision depends on agent i's response to her (expected) payment as well as the rival's response to firm i's R&D. Denoting her beliefs about firm j's R&D by $\widehat{x}_j$, principal i's problem becomes

$$\max_{\alpha_i, \beta_i, x_i} E\left\{\pi_i(\alpha_i, \beta_i, x_i; \widehat{x}_j) \mid \varepsilon_i, \varepsilon_j \in \Theta\right\} = E\left\{\Pi_i - w_i \mid \varepsilon_i, \varepsilon_j \in \Theta\right\}$$

$$\text{subject to} \;\; x_i^* = \arg\max_{x_i} E\left\{U_i\left(w_i, x_i\right) \mid \varepsilon_i \in \Theta\right\} \qquad (IC_i)$$

$$E\left\{U_i\left(w_i, x_i\right) \mid \varepsilon_i \in \Theta\right\} \geq 0 \qquad (IR_i)$$

The incentive compatibility constraint (IC_i) guarantees that agent i chooses the (expected) utility maximizing effort. The individual rationality constraint (IR_i) shows that agent i will participate in the R&D process only if her expected utility of doing so exceeds her reservation utility of zero. In lemma 1, we state that the certainty equivalence of agent i's utility can be expressed in a mean-variance form and the truncation of the distribution of the random terms does not affect the agent's optimal decision. The agent conducts the R&D level that would also be optimal if the distribution of the shocks was normal but not truncated.

Lemma 1 (Certainty equivalence of utility & truncated normal distribution) *If agent i has CARA preferences towards risk, linear contracts are used and the random terms follow a truncated normal distribution symmetric around the mean, then agent i's expected utility is given by*

$$E\left\{U_i\left(w_i, x_i\right) \mid \varepsilon_i \in \Theta\right\} = -\Phi_i e^{-r\left[\widetilde{U}_i(x_i)\right]} \text{ where } \widetilde{U}_i\left(x_i\right) = E\left(w_i\right) - \frac{r}{2} Var\left(w_i\right) - \psi(x_i),$$

and $\Phi_i \equiv \frac{\Phi\left(\frac{\theta+\sigma^2 r\beta_i}{\sigma}\right)-\Phi\left(\frac{-\theta+\sigma^2 r\beta_i}{\sigma}\right)}{\Phi\left(\frac{\theta}{\sigma}\right)-\Phi\left(\frac{-\theta}{\sigma}\right)}$. $\Phi\left(\frac{\theta}{\sigma}\right) - \Phi\left(\frac{-\theta}{\sigma}\right)$ *indicates the probability of* ε_i *falling into* Θ. *Given that* Φ_i *is positive and does not depend on effort, agent i's optimization problem is equivalent to choosing the* x_i *that maximizes the certainty equivalence of her utility* $\widetilde{U}_i\left(x_i\right)$.

Proof. See appendix. ■

Given that $\widetilde{U}_i\left(x_i\right) = \alpha_i + \beta_i x_i - \frac{r}{2}\beta_i^2\sigma^2 - \psi(x_i)$, the optimal effort x_i^* satisfies the first-order condition

$$\beta_i = \psi'\left(x_i^*\right). \tag{4}$$

To derive the optimal contractual parameters, we solve simultaneously both principals' constrained maximization problems and take the Kuhn-Tucker conditions. The IR_i constraint binds at the optimum and agents earn no rents: the fixed salary component, α_i, induces agent's participation at least cost. Thus, the optimal wage is $w_i\left(x_i\right) = \frac{r\sigma^2}{2}\left[\psi'\left(x_i\right)\right]^2 + \psi\left(x_i\right)$. Each agent is rewarded for the cost of effort she incurs and the risks she bears. By (3), we also have

$$E\left\{q_i^*\left(x_i, x_j\right)\right\} = \frac{1}{2+b}\left[A - \overline{c} + \frac{2x_i - bx_j}{2-b}\right], \tag{5}$$

implying that firms' R&D decisions are strategic substitutes. To guarantee that there exists an interior solution in this contracting/R&D game, we use the following Inada-type assumptions on the profit function π_i (i.e., Amir et al. (2000)):

$(A.1)$ $\frac{4[A(2-b)-2\overline{c}+b\theta]}{(4-b^2)^2} > \left[1 + r\sigma^2\psi''(0)\right]\psi'(0)(A.3)\ 4\left[A(2-b) - 2\overline{c} + b\theta\right]$

$(A.2)$ $\frac{4}{(2-b)^2(2+b)} < \left[1 + r\sigma^2\psi''(x)\right]\psi''(x)$ for all $x \in X$.

Assumption $(A.1)$ requires market demand to be high enough relative to the initial marginal cost and sets an upper bound on the marginal cost of effort at zero, so that each firm has incentives to undertake some R&D regardless of its rival's R&D choice. Assumption $(A.2)$ requires a strong form of convexity of the cost-of-effort function, so that the equilibrium of this game is unique. In particular, given that R&D decisions are strategic substitutes and thus the slope of R&D reaction functions is negative, $(A.2)$ guarantees that this slope is also higher than -1.[12,13]

Lemma 2 (Existence of unique interior equilibrium) *Under assumptions* $(A.1)-(A.2)$*, there exists a unique subgame perfect Nash equilibrium in R&D in the interior of the jointly undominated effective strategy space* X^2*.*

Proof. See appendix. ■

The optimal R&D level is

$$x^* = \frac{1}{4}\left(4-b^2\right)(2+b)\left[1+r\sigma^2\psi''(x^*)\right]\psi'(x^*) - (A-\overline{c}). \tag{6}$$

Risk plays a key role in the optimal decisions. Under full information, the principal extracts the complete rents via the base payment and agents' wage is equal to the marginal disutility of labor, $\psi'(.)$. However, under moral hazard, risk-aversion on the part of agents and uncertainty about performance induce the agents to seek insurance against low realizations of the R&D outputs and lower incentives are provided. Thus, effort falls short of its efficient level. In other words, the optimal effort decreases with risk, measured by $r\sigma^2$; i.e. $\frac{\partial x^*}{\partial(r\sigma^2)} < 0$ for all $x \in X$.[14]

3.3 Profits-risk relationship

The effect of risk on profits is not clear cut. In particular, firm i's equilibrium profits take the form $\pi^* = \Pi^* - \frac{r\sigma^2}{2}\left[\psi'(x^*)\right]^2 - \psi(x^*)$ where the R&D level is given by (6) and $\Pi^* = \frac{1}{(2+b)^2}\left(A-\overline{c}+x^*\right)^2$ by equation (5). Taking the derivative

$$\frac{\partial \pi^*}{\partial(r\sigma^2)} = \frac{2}{(2+b)^2}\left(A-\overline{c}+x^*\right)\frac{\partial x^*}{\partial(r\sigma^2)} - \left[\frac{1}{2}\psi'(x^*) + r\sigma^2\frac{\partial \psi'(x^*)}{\partial(r\sigma^2)}\right]\psi'(x^*) - \frac{\partial \psi(x^*)}{\partial(r\sigma^2)}, \tag{7}$$

we examine the underlying effects. First, the Cournot profits decrease with $r\sigma^2$: given that Π^* increases with efficiency-enhancing R&D and higher values of $r\sigma^2$ distort effort downwards, lower Cournot profits are realized as a result. Second, there are the direct and indirect effects on human

[12] Let the cost of effort function be $\frac{k}{2}x_i^2$ where higher k indicates lower efficiency. Assumption $(A.1)$ always holds and $(A.2)$ requires $\frac{4}{(2-b)^2(2+b)} < k\left(1+kr\sigma^2\right)$. Assumption $(A.2)$ also sufficies to guarantee that the sufficient condition of principal's problem holds.

[13] Provided that the assumptions hold for the extreme value θ, they also hold for the mean of the random terms, which is zero.

[14] Appendix $(A.3)$ provides a proof.

capital insurance. In particular, higher degrees of risk-aversion and uncertainty about performance induce the agent to seek additional insurance. In turn, the principal experiences lower residual profits. However, underprovision of R&D incentives due to risk-sharing also decreases the variable part of agent's compensation and thus, the variance of the payment; i.e., $r\sigma^2 \frac{\partial \psi'(x^*)}{\partial(r\sigma^2)} \psi'(x^*) < 0$. Agent i is induced to exert lower effort and thus, incurs lower risks. This (indirect) effect works in favor of the principal since she is required to provide less insurance. Third, there is the effect on the disutility of effort; i.e., $\frac{\partial \psi(x^*)}{\partial(r\sigma^2)} < 0$. The latter two effects capture the cost a firm saves by acquiring less R&D in response to higher risk.

We argue that there exists a regime in which cost-savings by providing lower-power R&D incentives due to moral hazard are substantial so that the profit-risk relationship turns out to be positive. This occurs when the gain in profit due to cost-savings exceeds the loss of profit due to lower market power a firm can possess by investing less in R&D. In this regime, the optimal profits increase as more insurance is provided. This result counters the prediction of the principal-agent theory where principals wish to have full information so as to perfectly monitor agents and achieve the optimal allocation of effort from their own perspective. We show that such motives can be reversed when firms compete aggressively in the product market.

Proposition 1 (Positive profits-risk relationship) *Under assumptions $(A.1)-(A.2)$, Cournot competitors' profits increase with risk if the cost of incentive provision is low and goods are close enough substitutes so that competition is stiff; i.e.,* $\frac{\partial \pi^*}{\partial(r\sigma^2)} > 0$ *if and only if* $\frac{4}{(4-b^2)(2+b)(1-b)} > [1 + r\sigma^2\psi''(x^*)]\,\psi''(x^*)$.

Proof. See appendix. ■

Proposition 1 states that, if effort exertion is not too costly (low $r\sigma^2$) and product market rivals compete (sufficiently) aggressively against each other (high b), they acquire such high levels of R&D so that they burn up their profits.[15] Under moral hazard, risk-sharing diminishes such appetite for innovation. Underprovision of incentives exactly due to the trade-off between effort exertion and insurance may generate considerable cost-savings and thus, higher profits for both rivals.[16] However, if $r\sigma^2$ exceeds the threshold specified in proposition 1, the cost of conducting R&D is so high that decreases profits.

[Figure 2 is about here]

This result indicates the desirability of the R&D rivals to adopt a collusive-like behavior in R&D so as to behave less aggressively in the product market. Thus, the separation between the business

[15] The product $\left(4-b^2\right)(2+b)(1-b)$ is decreasing in b.

[16] Suppose $\psi(x_i) = \frac{k}{2}x_i^2$; i.e. $\frac{\partial \pi^c}{\partial r\sigma^2} > 0$ if, and only if, $r\sigma^2 < \frac{1}{k}\left[\frac{4}{(4-b^2)(2+b)(1-b)k} - 1\right]$. Notably, for homogeneous-product duopolists, $b = 1$, the profits-risk relationship is positive for all parameter values; i.e. $\pi^c = \frac{(A-\bar{c})^2\left[9k\left(1+kr\sigma^2\right)-8\right]k\left(1+kr\sigma^2\right)}{\left[9k\left(1+kr\sigma^2\right)-4\right]^2}$ and $\frac{\partial \pi^c}{\partial r\sigma^2} = \frac{32(A-\bar{c})^2k^2}{\left[9k\left(1+kr\sigma^2\right)-4\right]^3} > 0$.

and research units, which implies the division between the ownership and control of R&D outputs under moral hazard, before firms meet in the market place, can be used as a commitment device for both rivals that softens their subsequent responses. Rivals will enjoy higher profits as a result.

This analysis gives new insights into firms' organizational structure. In industries where competition is low and R&D is costly, firm should adopt an organization structure that eliminates the information asymmetries about agents' actions. By doing so, firms will be able to manage the innovation process more efficiently and decrease incentive distortions. In contrast, much of the use of incentive pay could be in volatile industries, such as in high-technology industries and the financial sector. For instance, in microelectronics-based industries where competition is intensive, firms have strong incentives to mutually commit themselves to lower R&D levels. By delegating R&D decisions to a second party - i.e., a research team, an autonomous unit etc - or even by appointing more risk-averse researchers, rivals' strategic interactions are weakened and principals can become better-off.

4 R&D Incentives under spillovers

This section examines firms' R&D decisions in the presence of R&D spillovers. Spillovers (costlessly) decrease the rival's initial marginal cost creating a disincentive for the R&D taking firm. However, we argue that, if imperfect spillovers occur and firms' R&D decisions are strategic substitutes, spillovers can induce the product market competitors to intensify their R&D efforts in order to secure a cost advantage, decreasing their profits even further. Thus, we show that, as spillovers increase, there are additional benefits of using a self-control device that mutually decreases rivals' R&D efforts and allows for higher profits. To anticipate the effect of spillovers and competition on rivals' R&D, we decompose the R&D incentives and perform a comparative statics analysis.

4.1 Equilibrium

The R&D process is subject to cross-firm spillovers. As in D'Aspremont & Jacquemin (1990), agent i's R&D-output depends on the size of (unpaid) appropriation of its rival's research, hx_j. The R&D production function now takes the form

$$z_i = x_i + hx_j + \varepsilon_i. \tag{8}$$

The parameter h measures the spillover rate, $h \in [0, 1]$; i.e. the fraction of agent j's R&D that improves agent i's performance. That h is less than one and indicates the imperfect nature of R&D spillovers.

Agent i's compensation is now restricted to be linear to both agents' R&D-outputs since they are correlated due to spillovers (Holmström (1979), Holmström & Milgrom (1987), Holmström &

Tirole (1989)). Relative performance evaluations provide a richer information base on which to write contracts and allow each principal to better assess its agent's effort by looking at its rival's performance. The contract takes the form $(\alpha_i, \beta_i, \gamma_i)$ and agent i's receives

$$w_i = \alpha_i + \beta_i z_i + \gamma_i z_j, \tag{9}$$

where γ_i denotes the pay-for-rival performance parameter.[17] Thus, each agent's reward is conditioned on how well she performs compared to another.

The certainty equivalence of agent i's utility has as

$$\widetilde{U}_i(x_i) = \alpha_i + (\beta_i + h\gamma_i)\, x_i + (h\beta_i + \gamma_i)\, x_j - \frac{r}{2}\left[(\beta_i + h\gamma_i)^2 + (h\beta_i + \gamma_i)^2\right]\sigma^2 - \psi(x_i) \tag{10}$$

and the optimal effort is given by

$$\beta_i + h\gamma_i = \psi'\left(x_i^c\right) \tag{11}$$

The left hand side represents the "total" sensitivity of agent i's compensation to her own effort. In equilibrium, $\frac{\partial Var(w_i)/\partial \beta_i}{\partial \psi'\left(x_i^c\right)/\partial \beta_i} = \frac{\partial Var(w_i)/\partial \gamma_i}{\partial \psi'\left(x_i^c\right)/\partial \gamma_i}$ implying that each principal has two equivalent incentive tools available to use in order to affect agent's behavior. Given the concavity of the functions U_i and π_i in x_i, agents' CARA preferences and the (truncated) normality of the random terms, we can use the first-order approach and replace the IC_i constraint with equation (11).[18] By (3) and (8), we also have

$$E\left\{q_i^*\right\} = \frac{1}{2+b}\left[A - \bar{c} + \frac{(2-bh)\, x_i - (b-2h)\, x_j}{2-b}\right]. \tag{12}$$

The nature of R&D strategic interactions now depends on the sign of $b - 2h$. If spillovers are small enough, $h < \frac{b}{2}$, efforts are strategic substitutes, but if $h > \frac{b}{2}$, efforts are strategic complements. In the knife-edge case where $h = \frac{b}{2}$, each firm has a dominant strategy on R&D. To guarantee that a solution exists in this game, as in subsection 3.2, we make the following assumptions:

$(A.3)$ $\frac{2(2-bh)[A(2-b)-2\bar{c}(1-h)+(b-2h)\theta]}{(4-b^2)^2} > \left[1 + r\sigma^2\psi''(0)\right]\psi'(0)$

$(A.4)$ $\frac{2(2-bh)(A-\theta)}{(4-b^2)(2+b)} < \left[1 + r\sigma^2\psi''(\overline{x_i})\right]\psi'(\overline{x_i})$ where $\overline{x_i} = \frac{\bar{c}-\theta}{1+h}$

$(A.5)$ $\frac{2(2-bh)[2-bh+|b-2h|]}{(4-b^2)^2} < \left[1 + r\sigma^2\psi''(x)\right]\psi''(x)$ for all $x \in X$

[17] Lacetera & Zirulia (2012) consider effort to be multi-dimensional; i.e., effort for applied research and effort for basic research. Efforts are unobservable and unverifiable, while effort for basic research is assumed to be diffused. The marginal cost is non-contractible and the contracts are contingent on verifiable signals of both types of efforts. We consider a different framework where the principal-agent relationship is one-dimensional and the marginal cost is contractible.

[18] In a multi-agent framework, the monotone likelihood ratio property (MLRP) and the convexity of the distribution function condition (CDFC) are not sufficient for the first-order approach to be valid as in a single-agent setting. Itoh (1991) argues that, in a model with cross-agent interactions, a generalized CDFC for the joint probability distribution of the outputs is needed and the wage schemes must be nondecreasing. The coefficient of absolute risk aversion must also not decline too quickly. In our model, given the assumptions about the form of the contracts, the R&D production function and the agents' CARA preferences, the first-order approach applies.

Assumption $(A.3)$ insures that each firm finds it best to do some R&D. Assumption $(A.4)$ suffices to guarantee the interiority of the equilibrium. In particular, the feasibility lines $\overline{x_i} = \bar{c} - hx_j - \theta$ guarantee that the post-innovation marginal cost will be positive. Assumption $(A.4)$ requires the unit cost of doing R&D be large enough at the point where these lines intersect in order to moderate the R&D incentives. Assumption $(A.5)$ guarantees the uniqueness of this equilibrium.[19]

Proposition 2 (Spillovers & relative performance) *Under assumptions* $(A.3)-(A.5)$*, there exists a subgame perfect Nash equilibrium in performance-based parameters in which*

$$\beta^* = \psi'(x^*) \text{ and } \gamma^* = -h\beta^*,$$

where the optimal R&D-effort level x^* *has as*

$$x^* = \frac{1}{1+h}\left[\frac{(4-b^2)(2+b)\left[1+r\sigma^2\psi''(x^*)\right]\psi'(x^*)}{2(2-bh)} - (A-\bar{c})\right].$$

Proof. See appendix. ■

The positive sign of β^* indicates that higher own performance is compensated with a higher wage. In contrast, the principal sets γ^* to be negative, giving the agent a short position in rival's performance. The principal anticipates the positive contribution of spillovers on R&D performance and penalizes the agent when the rival does better. Such evaluation schemes can effectively be used as means of filtering out spillovers from agent's compensation. Thus, agent i's payment is no longer sensitive to j's R&D. Note also that the "compensation ratio", $\left|\frac{\gamma^*}{\beta^*}\right|$, is higher in compensation packages that are offered in industries with intensive spillovers.[20] The higher is h, the more valuable is the information contained in rival's R&D measure and thus, the use of relative performance evaluations becomes more essential.

4.2 Moral hazard & spillovers

The existence of spillovers changes the nature of R&D strategic interactions as well as the effects of moral hazard on firms' equilibrium profits. Corollary 1 states that a positive profits-risk relationship only exists if efforts are strategic substitutes and the cost of incentivizing the researchers to conduct R&D is small enough. If efforts are strategic complements, risk always decreases profits.[21]

Corollary 1 (Profits-risk relationship under spillovers) *Under assumptions* $(A.3)-(A.5)$*, in the presence of moral hazard, Cournot competitors' profits increase with risk,* $\frac{d\pi^*}{d(r\sigma^2)} > 0$*, if and only*

[19] Assumption $(A.3)$ is needed only if $h < \frac{b}{2}$ while $(A.4)$ plays a role only if $h > \frac{b}{2}$.

[20] Any compensation scheme that is a linear transformation of this cost-based contract will induce the same level of effort in equilibrium. For instance, agent's compensation could be contingent on outputs; i.e. $w_i = \alpha_i + \beta_i q_i + \gamma_i q_j$. In such a case, the optimal compensation ratio is $\left|-\frac{2h-b}{2-bh}\right|$. Note that the intensity of product market competition now affects the optimal incentive parameters.

[21] The proof of corollary 1 is similar to that of proposition 1 and provided in the online appendix.

if, efforts are strategic substitutes, $h < \frac{b}{2}$*, and*

$$\frac{2\left(2-bh\right)^{2}\left(1+h\right)}{\left(4-b^{2}\right)\left(2+b\right)\left[2\left(1-b\right)+h\left(4-b\right)\right]} > \left[1+r\sigma^{2}\psi''\left(x^{*}\right)\right]\psi''\left(x^{*}\right)$$

To interpret corollary 1, we first consider the underlying effects of R&D on firm's profits. Then, we examine how the optimal R&D incentives change with product market competition and spillovers. The decomposition of R&D incentives implies $\frac{\partial \pi_i}{\partial x_i} = \frac{\partial \Pi_i}{\partial q_j}\frac{\partial q_j}{\partial x_i} + \frac{\partial \Pi_i}{\partial c_i}\frac{\partial c_i}{\partial x_i} - \left[1+r\sigma^2\psi''(x_i)\right]\psi''(x_i)$. The direct effect of effort on profits comes through marginal cost reduction; i.e. the more a firm produces at a lower cost, the more it profits, $\frac{\partial \Pi_i}{\partial c_i}\frac{\partial c_i}{\partial x_i} = q_i$. This is the *scale effect*, which is positive. If $b > 0$, the indirect effects on firms' revenues are also at work; i.e. $\frac{\partial \Pi_i}{\partial q_j}\frac{\partial q_j}{\partial x_i} = \frac{\partial \Pi_i}{\partial q_j}\frac{1}{\Lambda}\left(\frac{\partial p_j}{\partial q_i} - 2h\frac{\partial p_i}{\partial q_i}\right) = \frac{b(b-2h)}{4-b^2}q_i$, where $\Lambda \equiv 4\frac{\partial p_i}{\partial q_i}\frac{\partial p_j}{\partial q_j} - \frac{\partial p_j}{\partial q_i}\frac{\partial p_i}{\partial q_j}$ is implied from the stability condition. First, there is the (positive) *strategic effect*: effort enhances the efficiency of production allowing the R&D-taking firm to produce more and increase its market share vis-à-vis its rival. Second, there is the (negative) *spillover effect*: agent i's effort also reduces firm j's initial marginal cost due to spillovers allowing the rival to be tougher in the product market. This effect is detrimental to the R&D-taking firm. The derivative $\frac{\partial \pi_i}{\partial x_i}$ shows the trade-off among all these effects against the increase in the cost of doing R&D. Rivals' R&D responses depend on the relative intensity of the strategic and spillover effect. Efforts are strategic substitutes if $h < \frac{b}{2}$ where the strategic effect dominates the spillover effect.

We attempt a Slutsky-like analysis to anticipate the effect of product market competition on R&D, We get $\frac{d(\partial \pi_i/\partial x_i)}{db} = 0 \Leftrightarrow \frac{\partial(\partial \pi_i/\partial x_i)}{\partial b} + \frac{\partial^2 \pi_i}{\partial x_i^2}\frac{dx^*}{db} = 0$. Assumption $(A.5)$ suffices to guarantee that the profit function is concave in x_i implying that $\frac{\partial^2 \pi_i}{\partial x_i^2} < 0$ for any $x_i \in X$. Thus, competition increases the optimal effort in the regime where it increases the marginal profitability of R&D. However, the sign of $\frac{\partial(\partial \pi_i/\partial x_i)}{\partial b}$ is ambiguous since competition intensifies the strategic and (negative) spillover effect while it makes the scale effect less important.

Corollary 2 (Effect of competition on effort) *Under assumptions* $(A.3) - (A.5)$*, the optimal R&D effort increases with product market competition,* $\frac{dx^c}{db} > 0$*, if, and only if,* $b > \frac{3+h-\left(9-2h-7h^2\right)^{\frac{1}{2}}}{2h}$.

Proof. See appendix. ■

Competition can increase R&D only if efforts are strategic substitutes. In this regime, in line with the literature, the relationship between competition and cost-reducing R&D investment is U-shaped; i.e., for low values of b, the derivative $\frac{\partial x^*}{\partial b}$ is negative, but it becomes positive when the condition in corollary 2 is satisfied. The intuition is as follows. On the one hand, the strategic effect increases with b: as demand becomes more elastic, a firm with a cost-advantage can more easily steal businesses from its rival. Thus, intensified competition increases the marginal benefit of cost reduction. On the other hand, the scale effect decreases: for higher b, the willingness to pay for firms' goods decreases, implying lower prices. In turn, a drop in output is required to compensate for the fall in profit. The (negative) spillover effect is also intensified. For higher b, a reduction in rival's marginal cost due to spillovers harms the R&D-taking firm by more. A lower-cost rival can more easily gain in market share. Therefore, competition strengthens R&D incentives when it makes the strategic effect more important than the other two effects, so that firms are driven by business stealing incentives.

Spillovers can also induce firms to invest more in R&D. Corollary 3 characterizes when this is so.

Corollary 3 (Effect of spillovers on effort) *Under assumptions* $(A.3)-(A.5)$*, the optimal R&D effort increases with spillovers; i.e.* $\frac{dx^*}{dh}>0$*, only if*

$$h<\frac{2-b}{2b} \text{ and } \frac{2(2-bh)^2(A-\overline{c})}{[2-b(1+2h)](4-b^2)(2+b)}<\left[1+r\sigma^2\psi''(x^*)\right]\psi'(x^*).$$

Proof. See appendix. ■

Spillovers intensify all three effects and thus, R&D increases when the positive strategic and scale effects become more important relative to the negative spillover effect. In particular, if efforts are strategic substitutes, $h<\frac{b}{2}$, a firm with lower cost can extend its businesses at the expense of its rival's. Thus, given the (imperfect) nature of spillovers, as h increases, firms provide higher-power R&D incentives in order to secure a cost advantage. This requires the cost of effort exertion to be relatively small.[22] Otherwise, if this cost exceeds the threshold specified in corollary 3, principals seem to be unwilling to bear such high R&D costs and exert lower effort in respond to higher h.

It all boils down to the following: if efforts are strategic substitutes and the cost of doing R&D is relatively small, the R&D activity is intensified when a firm is driven by business-stealing incentives due to the product market competition and attempts to secure that the cost-advantage will not be dissipated due to spillovers. In such a case, principals are eager to acquire more R&D and exert such a high level of R&D effort that harms them. In the presence of under moral hazard, providing insurance against the risk diminishes such incentives and cost-savings by conducting less R&D exactly due to risk-sharing may allow firms to realize higher profits in equilibrium.

In the regime where efforts are strategic complements, $h>\frac{b}{2}$, profits always decrease with risk. We first consider the monopoly case, $b=0$, where only the scale effect holds. Complementarities in R&D allow firms to exploit the spillovers only for efficiency enhancing (not strategic) reasons. There are mutual benefits from doing R&D. Thus, monopolists are always better-off under full information since they can perfectly monitor the agents. Less distorted decisions will enhance profits. For low R&D cost, this result applies even if $b>0$. However, if the R&D cost is high, risk decreases profits, but the intuition is different. Each firm has strong incentives to free-ride on its rival's research. Free-riding decreases effort and thus, further reduction in R&D due to risk-sharing will result in even lower profits.

[22] The free-rider's problem arises because, provided that firm j does not change its R&D level, an increase in h allows firm i to diminish its own R&D and appropriate j's R&D-output through spillovers.

4.3 Discussion and extensions

By analyzing the equilibrium R&D incentives, one can also argue that whenever competition increases equilibrium effort, as specified in corollary 2, the agency cost also increases. Similarly, whenever effort increases with spillovers, as specified in corollary 3, so does the agency cost.[23,24] In particular, under moral hazard, for higher $r\sigma^2$, the principals are less willing to exert effort as the contracts are 'rewritten' to accommodate intensified competition and spillovers. There is far more distortion in incentives. Effort responds less to an increase in b or h, and thus, the use of a self-commitment device that mutually weakens the incentives for effort exertion becomes increasingly more effective. The benefits from delegating the R&D decision under asymmetric information as a profit-enhancing decision are augmented.

This model can be extended in many ways and different directions. First, one can show that higher profits can be realized under moral hazard even if firms compete à la Bertrand. Firms' interactions in Bertrand and Cournot settings differ mainly in the following. For Bertrand competitors, the strategic effect is negative: cost-reducing R&D allows the R&D-taking firm to set a lower price. Competing for market share, the rival responds by cutting its own price too, implying lower profits for the innovator. Thus, product market competition gives rise only to detrimental effects on the innovator's profit. However, the cost-reducing incentives are also strong for Bertrand rivals exactly because firms behave so aggressively in the product market. Due to the anticipated price war and the imperfect nature of spillovers, rivals 'over'-invest in R&D, diminishing their profits. If firms operate under moral hazard, they innovate less and thus, price cutting as a market response becomes less profitable. As a result, risk-sharing can increase profits for both Bertrand and Cournot rivals.

A positive profit-risk relationship obtains only if firms compete simultaneously in both markets. In a sequential-move contracting/R&D game, the leader can solve the follower's problem and induce the follower to undertake the R&D level that maximizes the leader's profit. Consequently, the leader wishes to act under full information so as to effectively control the decisions taken by the agent and the follower.

It is also interesting to consider R&D incentives when the random terms are correlated. Suppose that the correlation coefficient is $\rho = \frac{\sigma_{ij}}{\sigma^2}$ where $|\rho| \leq 1$ and $\sigma_{ij} < 1 + r\sigma^2$. Positive or negative correlation is likely to occur when firms use similar or different technologies respectively. Due to the correlation of the random terms, the variance of the agent i's compensation becomes $Var\left(w_i\right) = \left[\left(\beta_i + h\gamma_i\right)^2 + \left(h\beta_i + \gamma_i\right)^2 + 2\rho\left(\beta_i + h\gamma_i\right)\left(h\beta_i + \gamma_i\right)\right]\sigma^2$. In our model where there are two forms of correlation between the agents' R&D-outputs, the optimal compensation ratio becomes $\left|\frac{\gamma^*}{\beta^*}\right| =$

[23] $\frac{\partial x^c}{\partial b} = -\frac{T[A-\bar{c}+(1+h)x^c]}{N}$ where $T \equiv \frac{4\left[3b-2-\left(2-b+b^2\right)h\right]}{(4-b^2)^2(2+b)}$, $N \equiv \frac{2(2-bh)(1+h)}{(4-b^2)(2+b)} - w''\left(x^c\right) < 0$. Given $\frac{\partial w''(x^c)}{\partial r\sigma^2} > 0$, $\frac{\partial(\partial x^c/\partial b)}{\partial(r\sigma^2)} = -\frac{T}{N^2}\left[\left(1+h\right)N\frac{\partial x^c}{\partial(r\sigma^2)} + \frac{\partial w''(x^c)}{\partial(r\sigma^2)}\left[A - \bar{c} + \left(1+h\right)x^c\right]\right]$ is negative wherever $\frac{\partial x^c}{\partial b} < 0 \Leftrightarrow T > 0$ by corollary 2.

[24] $\frac{\partial x^c}{\partial h} = -\frac{2}{N}\left(Mx^c - \Xi\right)$ where $M \equiv \frac{2-b(1+2h)}{(4-b^2)(2+b)}$, $\Xi \equiv \frac{b(A-\bar{c})}{(4-b^2)(2+b)} > 0$. Wherever $\frac{\partial x^c}{\partial h} > 0 \Leftrightarrow Mx^c - \Xi > 0$ by corollary 3, $\frac{\partial(\partial x^c/\partial h)}{\partial r\sigma^2} = -\frac{2}{N^2}\left[MN\frac{\partial x^c}{\partial r\sigma^2} + \frac{\partial w''(x^c)}{\partial r\sigma^2}\left(Mx^c - \Xi\right)\right]$ is negative.

$\left|-\frac{h+\rho}{1+\rho h}\right|$.

[Figure 3 is about here]

The optimal contract filters out both spillovers and the common shock from agent's reward. If $\rho > 0$, the principal sets γ^* negative since the agent acts in a favorable environment, which increases her performance. If $\rho < 0$, the sign of γ^* depends on the relative intensity of the two forms of correlation of R&D-outputs. γ^* is negative when spillovers matter more in agents' evaluations. However, if $h < |\rho|$, setting γ^* positive is a plausible way to encourage effort exertion. A principal incentivizes an agent to innovate by making her suffer less from a 'bad' outcome. Her reward now increases with a rival's performance. If $h = \rho$, then z_i becomes a sufficient statistic of x_i and agent i's compensation depends only on her own performance. In this setting with correlated random terms, rivals can also enjoy higher profits under moral hazard.

5 Conclusion

We examine researchers' incentives to carry out cost-reducing R&D in a setting with product market competition and R&D spillovers. Because R&D-inputs are not observable and the R&D process is subject to uncertainty over the R&D-outputs, moral hazard concerns and risk aversion of the agent become central. A linear principal-agent model is employed in which each principal is likely to offer a relative performance evaluation scheme whose performance measures are both own- and rival- firm cost reductions, reflecting that each agent 'appropriates' some part of its rival's research. This paper shows that compensation schemes based on explicit performance comparisons filter out spillovers from the reward packages by penalizing an agent when the rival performs better.

We argue that if R&D-efforts are strategic substitutes, then in highly competitive industries, firms innovate more in order to gain market share. If the cost of doing R&D is small, firms exert such a high level of effort that it decreases their profits. In such an environment, the existence of moral hazard can be profit-enhancing. In particular, the under-provision of incentives due to risk-sharing generates cost-savings and can increase firms' equilibrium profits. Thus, firms may prefer an organization structure where business and research teams are separated and agents abhor risk so as to use it as a collusive device that makes both firms better-off. Such results require research efforts to be strategic substitutes but can hold in both Cournot and Bertrand settings.

This analysis might be used to interpret some empirical evidence on the R&D performance of modern corporations in markets where innovation is rushed and knowledge is diffused. Science-based firms differ in behavior, management strategies and responses to market changes. This model also suggests avenues for future empirical research. The strategic nature of delegation and the use of incentive pay are themselves empirically testable. One could examine whether the organization structure of the firms and the form of the R&D contracts depend on the R&D and product market characteristics. The analysis suggests that there is a strategic motive stemming from the product market competition and R&D interactions. In addition, one could study firms' incentives to collaborate in the R&D market by forming R&D alliances, R&D joint ventures or by adopting any other form of collusive-like behavior in order to affect the intensity of competition in the downstream markets. Such decisions will differ for monopolists or less differentiated-product oligopolists. In high-tech industries where spillovers occur, this decision will also depend on whether the R&D choices are strategic complements or substitutes.

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A APPENDIX

A.1 Proof of lemma 1

By equations (1) and (2), agent i's expected utility has as[25]

$$E\left\{U_i\left(w_i,x_i\right)\mid \varepsilon_i\in\Theta\right\}=-e^{-r[\alpha_i+\beta_i x_i-\psi(x_i)]}E\left\{e^{-r\beta_i\varepsilon_i}\mid \varepsilon_i\in\Theta\right\}$$

The conditional density of ε_i has as

$$f\left(\varepsilon_i\mid\Theta\right)=\frac{\frac{1}{\sigma}\phi\left(\frac{\varepsilon_i}{\sigma}\right)}{\Phi\left(\frac{\theta}{\sigma}\right)-\Phi\left(\frac{-\theta}{\sigma}\right)},-\theta\leq\varepsilon_i\leq\theta \text{ where } \phi\left(\frac{\varepsilon_i}{\sigma}\right)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\varepsilon_i}{\sigma}\right)^2} \quad (13)$$

[25] It is also silently assumed that a positive constant term is added at agent's utility which moves this function upwards such that $E\left\{U_i\left(w_i,x_i\right)\mid\varepsilon_i\in\Theta\right\}\geq 0$.

$\Phi\left(\frac{\theta}{\sigma}\right) - \Phi\left(\frac{-\theta}{\sigma}\right)$ is the probability of ε_i falling into Θ. By equation (13), and letting $\widehat{r} = -r$, we have

$$\int_{-\theta}^{\theta} e^{\widehat{r}\beta_i\varepsilon_i} f(\varepsilon_i)\, d\varepsilon_i = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\theta}^{\theta} e^{\widehat{r}\beta_i\varepsilon_i} e^{-\frac{1}{2}\left(\frac{\varepsilon_i}{\sigma}\right)^2} d\varepsilon_i = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\theta}^{\theta} e^{-\frac{\left(\varepsilon_i^2 - 2\sigma^2\widehat{r}\beta_i\varepsilon_i\right)}{2\sigma^2}} d\varepsilon_i =$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\theta}^{\theta} e^{-\frac{\left[\varepsilon_i^2 - 2\varepsilon_i\left(\sigma^2\widehat{r}\beta_i\right) + \left(\sigma^2\widehat{r}\beta_i\right)^2 - \left(\sigma^2\widehat{r}\beta_i\right)^2\right]}{2\sigma^2}} d\varepsilon_i = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\theta}^{\theta} e^{-\frac{\left(\varepsilon - \sigma^2\widehat{r}\beta_i\right)^2}{2\sigma^2}} e^{\frac{\left(\sigma^2\widehat{r}\beta_i\right)^2}{2\sigma^2}} d\varepsilon_i =$$

$$= e^{\frac{\widehat{r}^2\beta_i^2\sigma^2}{2}} \frac{1}{\sigma\sqrt{2\pi}} \int_{-\theta}^{\theta} e^{-\frac{\left(\varepsilon_i - \sigma^2\widehat{r}\beta_i\right)^2}{2\sigma^2}} d\varepsilon_i = e^{\frac{\widehat{r}^2\beta_i^2\sigma^2}{2}} \int_{-\theta}^{\theta} \frac{1}{\sigma}\phi\left(\frac{\varepsilon_i - \sigma^2\widehat{r}\beta_i}{\sigma}\right) d\varepsilon_i =$$

$$= e^{\frac{\widehat{r}^2\beta_i^2\sigma^2}{2}} \left[\Phi\left(\frac{\theta - \sigma^2\widehat{r}\beta_i}{\sigma}\right) - \Phi\left(\frac{-\theta - \sigma^2\widehat{r}\beta_i}{\sigma}\right)\right]$$

$$\text{Thus, } E\left\{e^{\widehat{r}\beta_i\varepsilon_i} \mid \varepsilon_i \in \Theta\right\} = \frac{\int_{-\theta}^{\theta} e^{\widehat{r}\beta_i\varepsilon_i} f(\varepsilon_i)\, d\varepsilon_i}{\Phi\left(\frac{\theta}{\sigma}\right) - \Phi\left(\frac{-\theta}{\sigma}\right)} = e^{\frac{\widehat{r}^2\beta_i^2\sigma^2}{2}} \frac{\Phi\left(\frac{\theta - \sigma^2\widehat{r}\beta_i}{\sigma}\right) - \Phi\left(\frac{-\theta - \sigma^2\widehat{r}\beta_i}{\sigma}\right)}{\Phi\left(\frac{\theta}{\sigma}\right) - \Phi\left(\frac{-\theta}{\sigma}\right)} = e^{\frac{\widehat{r}^2\beta_i^2\sigma^2}{2}} \Phi_i$$

where $\Phi_i = \frac{\Phi\left(\frac{\theta - \sigma^2\widehat{r}\beta_i}{\sigma}\right) - \Phi\left(\frac{-\theta - \sigma^2\widehat{r}\beta_i}{\sigma}\right)}{\Phi\left(\frac{\theta}{\sigma}\right) - \Phi\left(\frac{-\theta}{\sigma}\right)}$. We get $E\left\{U_i(w_i, x_i) \mid \varepsilon_i \in \Theta\right\} = -\Phi_i e^{-r\left[\alpha_i + \beta_i x_i - \frac{r}{2}\beta_i^2\sigma^2 - \psi(x_i)\right]}$. Given that Φ_i is positive and independent of x_i, agent i's optimization problem is equivalent to choose

$$x_i \in \arg\max \widetilde{U}_i(x_i) \text{ where } \widetilde{U}_i(x_i) = \alpha_i + \beta_i x_i - \frac{r}{2}\beta_i^2\sigma^2 - \psi(x_i)$$

$\widetilde{U}_i(x_i)$ is the certainty equivalence of agent i's utility. Thus, given CARA preferences and linear contracts, agent i's problem has a closed form solution even if ε_i follows a *truncated* normal distribution, which is symmetric around the mean.

A.2 Proof of lemma 2

The existence of an equilibrium in R&D requires to show that the R&D reaction functions - denoted by $r_i(x_j)$ - are (monotone) contractions and apply the Contraction Mapping Theorem. Assumption $(A.2)$ guarantees that π_i is strictly concave in x_i and thus, $r_i(x_j)$ is single-valued and continuous. Given also that the action set X is compact, an equilibrium exists. The interiority of this equilibrium requires π_i to be strictly increasing in a neighborhood of $x_i = 0$ for all $x_j \in X$; i.e. $\frac{\partial \pi_i(0, x_j)}{\partial x_i} = \left[A - \overline{c} - \frac{bx_j}{2-b}\right] \frac{4}{(4-b^2)(2+b)} - \left[1 + r\sigma^2\psi''(0)\right]\psi'(0) > 0$. If this inequality holds for $x_j = \overline{c} - \theta$, it will also hold for all $x_j \in X$ which is guaranteed by assumption $(A.1)$. $r_i(x_j)$ must also be strictly decreasing; i.e. $\frac{d(\partial \pi_i/\partial x_i)}{dx_j} = 0 \Leftrightarrow \frac{\partial^2 \pi_i(r_i(x_j), x_j)}{\partial x_i \partial x_j} + \frac{\partial^2 \pi_i(r_i(x_j), x_j)}{\partial x_i^2} r_i'(x_j) = 0$ implying

$$r_i'(x_j) = -\frac{\frac{\partial^2 \pi_i(r_i(x_j), x_j)}{\partial x_i \partial x_j}}{\frac{\partial^2 \pi_i(r_i(x_j), x_j)}{\partial x_i^2}} = \frac{\frac{4b}{(4-b^2)^2}}{\frac{8}{(4-b^2)^2} - \left[1 + r\sigma^2\psi''(x_i)\right]\psi''(x_i)} = -\frac{4b}{(4-b^2)^2\left[1 + r\sigma^2\psi''(x_i)\right]\psi''(x_i) - 8} \tag{14}$$

Assumption $(A.2)$ guarantees that $r_i'(x_j)$ is negative. The uniqueness of this equilibrium requires $-1 < r_i'(x_j) < 1$ for all $x_j \in X$. Given that $r_i'(x_j) < 0$, it suffices to show that $-1 < r_i'(x_j)$ which is implied by assumption $(A.2)$. Therefore, given that the game in the product market has a unique equilibrium, the subgame perfect equilibrium of the overall game is also unique.

A.3 Proof of proposition 1

We first need to examine the effect of $r\sigma^2$ on the optimal effort level; i.e. $\frac{d(\partial\pi_i/\partial x_i)}{d(r\sigma^2)} = 0 \Leftrightarrow \frac{\partial(\partial\pi_i/\partial x_i)}{\partial(r\sigma^2)} + \frac{\partial^2\pi_i}{\partial x_i^2}\frac{dx^c}{d(r\sigma^2)} = 0$ where

$$\frac{\partial^2\pi_i}{\partial x_i^2} = \frac{8}{(4-b^2)^2} - \left[1 + r\sigma^2\psi''(x_i)\right]\psi''(x_i) \tag{15}$$

Assumption $(A.2)$ guarantees that the profit function is concave in x_i - i.e. $\frac{\partial^2\pi_i}{\partial x_i^2} < 0$ for all $x_i \in X$. Thus, we consider the effect of risk on the marginal profitability of R&D. We substitute x_j in the first-order condition $\frac{4}{4-b^2}q_i^c - [1 + r\sigma^2\psi''(x_i)]\,\psi'(x_i) = 0$, where q_i^c is given by (5), with the optimal value x^c and differentiate with respect to $r\sigma^2$, taking x_i as constant. We obtain

$$\frac{\partial(\partial\pi_i/\partial x_i)}{\partial(r\sigma^2)} = -\frac{4b}{(4-b^2)^2}\frac{dx^c}{d(r\sigma^2)} - \psi''(x^c)\,\psi'(x^c) \tag{16}$$

Having (15) and (16), the decomposition becomes

$$-\psi''(x^c)\,\psi'(x^c) + \left[\frac{4}{(4-b^2)(2+b)} - \left[1 + r\sigma^2\psi''(x^c)\right]\psi''(x^c)\right]\frac{dx^c}{d(r\sigma^2)} = 0 \tag{17}$$

The cost of effort function $\psi(x_i)$ is convex and the term in the brackets is negative by assumption $(A.2)$ which imply that $\frac{dx^c}{d(r\sigma^2)} < 0$ for all $x_i \in X$. Thus, from (7), π^c increases with $r\sigma^2$ if, and only if,

$$\frac{2}{(2+b)^2}(A - \overline{c} + x^c)\frac{dx^c}{d(r\sigma^2)} > \left[\frac{1}{2}\psi'(x^c) + r\sigma^2\frac{\partial\psi'(x^c)}{\partial(r\sigma^2)} + \frac{\partial x^c}{\partial(r\sigma^2)}\right]\psi'(x^c) \tag{18}$$

By equation (6), we get $\frac{2}{(2+b)^2}[A - \overline{c} + x^c] = \frac{2-b}{2}[1 + r\sigma^2\psi''(x^c)]\,\psi'(x^c)$ and by equation (17), we obtain $\frac{dx^c}{d(r\sigma^2)} = -\frac{(4-b^2)(2+b)\psi''(x^c)\psi'(x^c)}{(4-b^2)(2+b)[1+r\sigma^2\psi''(x^c)]\psi''(x^c)-4}$. Having also $\frac{\partial\psi'(x^c)}{\partial(r\sigma^2)} = \psi''(x^c)\frac{dx^c}{d(r\sigma^2)}$, inequality (18) becomes

$$\frac{[1 + r\sigma^2\psi''(x^c)]\,\psi''(x^c)}{[1 + r\sigma^2\psi''(x^c)]\,\psi''(x^c) - \frac{4}{(4-b^2)(2+b)}} > \frac{1}{b}, \tag{19}$$

which implies the condition in proposition 1.

A.4 Proof of proposition 2

In the presence of spillovers, for a unique interior equilibrium to exist, additional conditions are required. In particular, the interiority of the equilibrium requires, if $h < \frac{b}{2}$, π_i to be strictly increasing in a neighborhood of $x_i = 0$ for all $x_j \in X$; i.e. $\frac{\partial\pi_i(0,x_j)}{\partial x_i} = \left[A - \overline{c} + \frac{(2h-b)x_j}{2-b}\right]\frac{2(2-bh)}{(4-b^2)(2+b)} - [1 + r\sigma^2\psi''(0)]\,\psi'(0) > 0$. If this inequality holds for $x_j = \overline{c} - \theta$, it will also hold for all $x_j \in X$ since $2h - b < 0$. This is guaranteed by assumption $(A.1)$. $r_i(x_j)$ must also be strictly decreasing in

$\left[0, \overline{c} - \overline{h}x_j\right]$. We have

$$r_i'(x_j) = -\frac{\frac{\partial^2 \pi_i(r_i(x_j), x_j)}{\partial x_i \partial x_j}}{\frac{\partial^2 \pi_i(r_i(x_j), x_j)}{\partial x_i^2}} = -\frac{\frac{2(2-bh)(2h-b)}{(4-b^2)^2}}{\frac{2(2-bh)^2}{(4-b^2)^2} - \left[1 + r\sigma^2 \psi''(x_i)\right] \psi''(x_i)} = \frac{2(2-bh)(2h-b)}{(4-b^2)^2 \left[1 + r\sigma^2 \psi''(x_i)\right] \psi''(x_i) - 2(2-bh)^2} \tag{20}$$

which is negative in this regime by assumption $(A.3)$. If $h > \frac{b}{2}$, it suffices to show that π_i is strictly decreasing in x_i at the point where the feasibility lines intersect. This is where $\overline{x_i} = \overline{x_j} = \frac{\overline{c} - \theta}{1+h}$. Assumption $(A.2)$ guarantees that $\frac{\partial \pi_i(\overline{x_i}, \overline{x_j})}{\partial x_i} < 0$. In this regime, equation (20) is also positive implying that $r_i(x_j)$ is strictly increasing whenever interior.

For this equilibrium to be unique, it suffices to show that $-1 < r_i'(x_j) < 1$ for all $x_j \in X$. If $h < \frac{b}{2}$, given that $r_i'(x_j) < 0$, it suffices to show that $-1 < r_i'(x_j)$. This requires $\left[1 + r\sigma^2 \psi''(x_i)\right] \psi''(x_i) > \frac{2(2-bh)[b-2h+2-bh]}{(4-b^2)^2}$ which is implied by assumption $(A.3)$. If $h > \frac{b}{2}$, given that $r_i'(x_j) > 0$, it suffices to show that $1 > r_i'(x_j)$. This requires $\left[1 + r\sigma^2 \psi''(x_i)\right] \psi''(x_i) > \frac{2(2-bh)[2h-b+2-bh]}{(4-b^2)^2}$ which is also implied by assumption $(A.3)$.

To derive the equilibrium, we simplify things by rearranging the terms of agent i's compensation in equation (9) and denoting $\varphi_{ii} \equiv \beta_i + h\gamma_i$, $\varphi_{ij} \equiv h\beta_i + \gamma_i$ the coefficients of x_i, x_j respectively; i.e. agent i's expected wage becomes $E\{w_i\} = \alpha_i + \varphi_{ii}x_i + \varphi_{ij}x_j$. Given (10) and (11), the Lagrange function of principal i's problem has as

$$L_i = q_i^2 - \left(\alpha_i + \varphi_{ii}x_i + \varphi_{ij}x_j\right) + \lambda_i \left[\varphi_{ii} - \psi'(x_i)\right] + \mu_i \left[\alpha_i + \varphi_{ii}x_i + \varphi_{ij}x_j - \frac{1}{2} r \left(\varphi_{ii}^2 + \varphi_{ij}^2\right) \sigma^2 - \psi(x_i)\right]$$

where q_i is given by (12). Omitting details, the Kuhn-Tucker condition with respect to α_i gives $-1 + \mu_i = 0 \Leftrightarrow \mu_i = 1$ implying that the (IR_i) constraint is binding at the optimum. Then, the Kuhn-Tucker conditions of i's and j's problem have respectively as

$$\begin{aligned}
\frac{\partial L_i}{\partial \lambda_i} &= \varphi_{ii} - \psi'(x_i) \geq 0 \text{ or } \lambda_i \geq 0 \text{ such that } \lambda_i \left[\varphi_{ii} - \psi'(x_i)\right] = 0, \forall i \\
\frac{\partial L_i}{\partial \beta_i} &= -\frac{1}{2} r \left[2\varphi_{ii} \frac{\partial \varphi_{ii}}{\partial \beta_i} + 2\varphi_{ij} \frac{\partial \varphi_{ij}}{\partial \beta_i}\right] \sigma^2 + \lambda_i \frac{\partial \varphi_{ii}}{\partial \beta_i} \leq 0 \text{ or } \beta_i \geq 0 \text{ such that } \frac{\partial L_i}{\partial \beta_i} \beta_i = 0, \forall i \\
\frac{\partial L_i}{\partial \gamma_i} &= -\frac{1}{2} r \left[2\varphi_{ii} \frac{\partial \varphi_{ii}}{\partial \gamma_i} + 2\varphi_{ij} \frac{\partial \varphi_{ij}}{\partial \gamma_i}\right] \sigma^2 + \lambda_i \frac{\partial \varphi_{ii}}{\partial \gamma_i} \geq 0 \text{ or } \gamma_i \leq 0 \text{ such that } \frac{\partial L_i}{\partial \gamma_i} \gamma_i = 0, \forall i \\
\frac{\partial L_i}{\partial x_i} &= \frac{2(2-bh)}{4-b^2} q_i - \lambda_i \psi''(x_i) - \psi'(x_i) \leq 0 \text{ or } x_i \geq 0 \text{ such that } \frac{\partial L_i}{\partial x_i} x_i = 0, \forall i
\end{aligned}$$

Under assumptions $(A.3)$-$(A.5)$, both firms innovate, $x_i > 0$, $x_j > 0$, and $\lambda_i > 0$, $\lambda_j > 0$, implying respectively

$$\varphi_{ii} = \psi'(x_i), \forall i$$

$$-r \left[\varphi_{ii} \frac{\partial \varphi_{ii}}{\partial \beta_i} + \varphi_{ij} \frac{\partial \varphi_{ij}}{\partial \beta_i}\right] \sigma^2 + \lambda_i \frac{\partial \varphi_{ii}}{\partial \beta_i} = 0, \forall i \tag{21}$$

$$-r \left[\varphi_{ii} \frac{\partial \varphi_{ii}}{\partial \gamma_i} + \varphi_{ij} \frac{\partial \varphi_{ij}}{\partial \gamma_i}\right] \sigma^2 + \lambda_i \frac{\partial \varphi_{ii}}{\partial \gamma_i} = 0, \forall i \tag{22}$$

$$\frac{2(2-bh)}{4-b^2}q_i - \lambda_i\psi''(x_i) - \psi'(x_i) = 0, \forall i \tag{23}$$

Dividing equations (21) and (22), we get $\varphi_{ij}\left(\frac{\partial\varphi_{ij}}{\partial\beta_i}\frac{\partial\varphi_{ii}}{\partial\gamma_i} - \frac{\partial\varphi_{ij}}{\partial\gamma_i}\frac{\partial\varphi_{ii}}{\partial\beta_i}\right) = 0$. Given the forms of φ_{ii} and φ_{ij}, the term in the parenthesis is equal to -1, implying that $\varphi_{ij} = 0 \Leftrightarrow \gamma_i = -h\beta_i$ and $\varphi_{ii} = \psi'(x_i) = \beta_i$. By equation (21), we also obtain that $\lambda_i = r\sigma^2\psi'(x_i)$. Thus, x^c solves the equation (23) which implies proposition 2.

A.5 Proof of corollary 2

To anticipate the effects of b on the marginal profitability of R&D, as in subsection ($A.3$), we substitute x_j in equation (23) with the optimal value x^c, differentiate with respect to b, taking x_i as constant, and then, substitute x_i with x^c. We get

$$\frac{\partial(\partial\pi_i/\partial x_i)}{\partial b} = \frac{2[4b-h(4+b^2)]}{(4-b^2)^2(2+b)}[A-\bar{c}+(1+h)x^c] + \frac{2(2-bh)}{4-b^2}\left[-\frac{A-\bar{c}}{(2+b)^2} + \frac{1+h}{(2-b)^2}x^c + \frac{2h-b}{4-b^2}\right]$$

Given also that $\frac{\partial^2\pi_i}{\partial x_i^2} = \frac{2(2-bh)^2}{(4-b^2)^2} - [1+r\sigma^2\psi''(x^c)]\psi''(x^c)$, the decomposition $\frac{\partial(\partial\pi_i/\partial x_i)}{\partial b} + \frac{\partial^2\pi_i}{\partial x_i^2}\frac{dx^c}{db} = 0$ gives

$$\frac{4[3b-2-(2-b+b^2)h][A-\bar{c}+(1+h)x^c]}{(4-b^2)^2(2+b)} + \left[\frac{2(2-bh)(1+h)}{(4-b^2)(2+b)} - \left[1+r\sigma^2\psi''(x^c)\right]\psi''(x^c)\right]\frac{\partial x^c}{\partial b} = 0 \tag{24}$$

The coefficient of $\frac{\partial x^c}{\partial b}$ is negative by assumption ($A.3$). Thus, the derivative $\frac{dx^c}{db}$ is positive if, and only if, the first term in equation (24) is also positive. This requires $3b-2-(2-b+b^2)h > 0$, and thus, $b > \frac{3+h-(9-2h-7h^2)^{1/2}}{2h}$. By the l'Hôpital rule, it is $\lim_{h\to 0^+}\left\{\frac{3+h-(9-2h-7h^2)^{1/2}}{2h}\right\} = \lim_{h\to 0^+}\left\{\frac{1}{2}\left(1+\frac{1+7h}{[(1-h)(9+7h)]^{1/2}}\right)\right\} = \frac{2}{3}$. Note that, for $h > 0$, competition can increase R&D only if efforts are strategic substitutes; i.e. let $2h \leq \frac{3+h-(9-2h-7h^2)^{1/2}}{2h} \Leftrightarrow \frac{3+h-(9-2h-7h^2)^{1/2}-4h^2}{2h} \geq 0$ which is true for all $h \in (0,1]$ and $\lim_{h\to 0^+}\left\{\frac{3+h-(9-2h-7h^2)^{1/2}-4h^2}{2h}\right\} = \lim_{h\to 0^+}\left\{\frac{1}{2}\left(1-2h+\frac{1+7h}{[(1-h)(9+7h)]^{1/2}}\right)\right\} = \frac{2}{3}$.

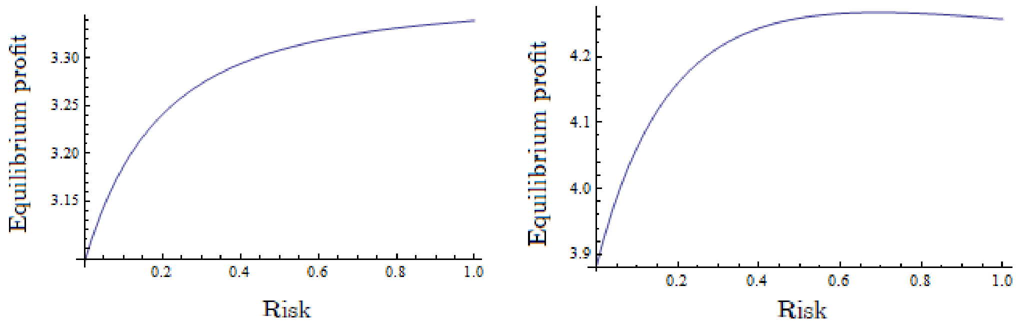
A.6 Proof of corollary 3

Given that $\frac{\partial(\partial\pi_i/\partial x_i)}{\partial h} = -\frac{2b[A-\bar{c}+(1+h)x^c]}{(4-b^2)(2+b)} + \frac{2(2-bh)}{4-b^2}\left[\frac{1}{2+b}x^c + \frac{2h-b}{4-b^2}\frac{\partial x^c}{\partial h}\right]$, the decomposition of the effects of spillovers implies

$$2\frac{[2-b(1+2h)]x^c - b(A-\bar{c})}{(4-b^2)(2+b)} + \left[\frac{2(2-bh)(1+h)}{(4-b^2)(2+b)} - \left[1+r\sigma^2\psi''(x^c)\right]\psi''(x^c)\right]\frac{dx^c}{dh} = 0 \tag{25}$$

Assumption ($A.5$) guarantees that the coefficient of $\frac{dx^c}{dh}$ is negative and thus, the derivative $\frac{dx^c}{dh}$ is positive only if, the first term in equation (25) is also positive. This is so when the following two conditions hold: i.e. $2-b(1+2h) > 0 \Leftrightarrow h < \frac{2-b}{2b}$ and $\frac{b(A-\bar{c})}{2-b(1+2h)} < x^c$ where x^c satisfies equation (6). Note that the first condition is not restrictive if efforts are strategic substitutes, $h < \frac{b}{2}$, since

$\frac{b}{2} \leq \frac{2-b}{2b} \Leftrightarrow 2 - b \geq b^2$ for any $b \in [0, 1]$.



2.1 2.2

Figure 2 shows the profits-risk relationship when the cost-of-effort function is quadratic of the form $\psi(x_i) = \frac{k}{2}x_i^2$, $A = 10$ and $\bar{c} = 4.5$. Figure 2.1 assumes that $b = 1$, $k = 2$ and figure 2.2 assumes that $b = 0.75$, $k = 1$.

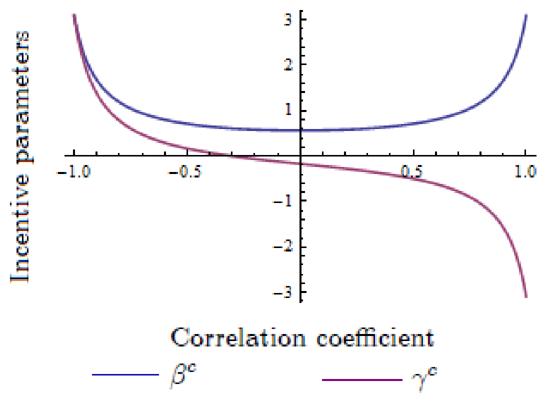


Figure 3 shows the optimal incentive parameters, β^c and γ^c, when the random terms are correlated, the cost-of-effort function is $\psi(x_i) = \frac{k}{2}x_i^2$ and $A = 10$, $\bar{c} = 4.5$, $h = 0.3$, $b = 1$, $k = 1.5$, $r\sigma^2 = 2$.