Supplementary material (Appendix B):

"Incentives to help or sabotage among co-workers"

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B.1. The multi-period model

Suppose that the manager and the workers interact for $T > 2$ periods. The conditional expectation of worker i's ability in period t, given the history of realized outputs, where $Z_{t-1}^i \equiv \left(z_1^i, ..., z_{t-1}^i\right)$ and $Z_{t-1}^j \equiv \left(z_1^j, ..., z_{t-1}^j\right)$, is

$$E\left\{\theta^i \mid Z_{t-1}^i, Z_{t-1}^j\right\} = \rho_{t-1}^{ii} \sum_{\tau=1}^{t-1} \left(z_\tau^i - \widehat{e}_\tau^i - k_j \widehat{a}_\tau^j\right) + \rho_{t-1}^{ij} \sum_{\tau=1}^{t-1} \left(z_\tau^j - \widehat{e}_\tau^j - k_i \widehat{a}_\tau^i\right).$$

The (conditional) correlation coefficients of θ^i with z_{t-1}^i and z_{t-1}^j, respectively, are

$$\begin{aligned} \rho_{t-1}^{ii} &= \frac{\sigma_i^2}{\lambda_{t-1}} \left[\varphi_i^2 + (t-1)(1-h_i h_j)\sigma_j^2\right], \\ \rho_{t-1}^{ij} &= \frac{\sigma_i^2}{\lambda_{t-1}} \left[h_i \varphi_j^2 - (t-1)(1-h_i h_j) h_j \sigma_j^2\right], \end{aligned}$$

where $\lambda_{t-1} \equiv \varphi_i^2 \varphi_j^2 + (\tau-1)^2 (1-h_i h_j)^2 \sigma_i^2 \sigma_j^2 + (\tau-1)\left[\varphi_i^2\left(\sigma_j^2 + h_i^2 \sigma_i^2\right) + \varphi_j^2\left(\sigma_i^2 + h_j^2 \sigma_j^2\right)\right]$. Consider a sequence of contracts with pay-for-performance parameters $\left(\beta_1^i, \gamma_1^i, ..., \beta_T^i, \gamma_T^i\right)$. A worker i maximizes

$$CE^i = \sum_{t=1}^{T} \left[E\left\{w_t^i \mid Z_{t-1}^i, Z_{t-1}^j\right\} - g\left(e_t^i\right) - y\left(a_t^i\right)\right] - \frac{r_i}{2} var\left\{\sum_{t=1}^{T} w_t^i \mid Z_{t-1}^i, Z_{t-1}^j\right\}.$$

Agent i's optimal effort satisfies

$$\beta_t^i + \sum_{\tau=t+1}^{T} M_\tau^{ii} = g'\left(e_t^{i*}\right) \text{ and } k_i\left(\gamma_t^i + \sum_{\tau=t+1}^{T} M_\tau^{ij}\right) = y'\left(a_t^{i*}\right), \tag{1}$$

$$\text{where} \quad \begin{aligned} M_\tau^{ii} &= \left(1 + h_i - \beta_\tau^{i*} - h_i \gamma_\tau^{i*}\right) \rho_{\tau-1}^{ii} - \left(h_j \beta_\tau^{i*} + \gamma_\tau^{i*}\right) \rho_{\tau-1}^{ji}, \\ M_\tau^{ij} &= \left(1 + h_i - \beta_\tau^{i*} - h_i \gamma_\tau^{i*}\right) \rho_{\tau-1}^{ij} - \left(h_j \beta_\tau^{i*} + \gamma_\tau^{i*}\right) \rho_{\tau-1}^{jj}. \end{aligned} \tag{2}$$

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Letting $B_t^i \equiv \beta_t^i + \sum_{\tau=t+1}^T M_\tau^{ii}$ and $\Gamma_t^i \equiv \gamma_t^i + \sum_{\tau=t+1}^T M_\tau^{ij}$, the variance of the wages is

$$var\left\{\sum_{\tau=t}^T w_\tau^i\right\} = \left[\sum_{\tau=t}^T \left(B_\tau^i + h_j\Gamma_\tau^i\right)\right]^2 \sigma_{i,t}^2 + \left[\sum_{\tau=t}^T \left(h_j B_\tau^i + \Gamma_\tau^i\right)\right]^2 \sigma_{j,t}^2 + \sum_{\tau=t}^T \left[\left(B_\tau^i\right)^2 \varphi_i^2 + \left(\Gamma_\tau^i\right)^2 \varphi_j^2\right],$$

where $\sigma_{i,t}^2 = \left[1-(t-1)\left(\rho_{\tau-1}^{ii} + h_i\rho_{\tau-1}^{ij}\right)\right]\sigma_i^2$. The Kuhn-Tucker conditions with respect to β_t^{i*} and γ_t^{i*} give

$$\begin{aligned}
\sigma_{i,t}^2 \sum_{\tau=t}^T \left(B_\tau^i + h_i\Gamma_\tau^i\right) + h_j\sigma_{j,t}^2 \sum_{\tau=t}^T \left(h_j B_\tau^i + \Gamma_\tau^i\right) + B_t^i\varphi_i^2 &= \frac{\lambda_i}{r_i}, \\
h_i\sigma_{i,t}^2 \sum_{\tau=t}^T \left(B_\tau^i + h_i\Gamma_\tau^i\right) + \sigma_{j,t}^2 \sum_{\tau=t}^T \left(h_j B_\tau^i + \Gamma_\tau^i\right) + \Gamma_t^i\varphi_j^2 &= \frac{k_i\mu_i}{r_i}.
\end{aligned}$$

Solving with respect to β_t^{i*} and γ_t^{i*}, we obtain

$$\begin{aligned}
\beta_t^{i*} &= \frac{1}{\Omega_t^i} - M_\tau^{ii} - r_i g_t^{i\prime\prime} \frac{\left(k_i^2 + r_i\varphi_j^2 y_t^{i\prime\prime}\right)\left[\left(\sigma_{i,t}^2 + h_j^2\sigma_{j,t}^2\right)\left(\sum_{\tau=t+1}^T \beta_\tau^{i*}\right) + \Sigma_t^{ij}\left(\sum_{\tau=t+1}^T \gamma_\tau^{i*}\right)\right]}{\zeta_t^i\Omega_t^i} \\
&\quad - r_i g_t^{i\prime\prime} \frac{r\left(1-h_ih_j\right)^2 \sigma_{i,t}^2\sigma_{j,t}^2 g_t^{i\prime\prime}\left(\sum_{\tau=t+1}^T \beta_\tau^{i*}\right)}{\zeta_t^i\Omega_t^i},
\end{aligned}$$

$$\begin{aligned}
\gamma_t^{i*} &= \frac{\Phi_t^i}{\Omega_t^i} - M_\tau^{ij} - r_i y_t^{i\prime\prime} \frac{\left(1 + r_i\varphi_i^2 g_t^{i\prime\prime}\right)\left[\Sigma_t^{ij}\left(\sum_{\tau=t+1}^T \beta_\tau^{i*}\right) + \left(\sigma_{j,t}^2 + h_i^2\sigma_{i,t}^2\right)\left(\sum_{\tau=t+1}^T \gamma_\tau^{i*}\right)\right]}{\zeta_t^i\Omega_t^i} \\
&\quad - r_i y_t^{i\prime\prime} \frac{r_i\left(1-h_ih_j\right)^2 \sigma_{i,t}^2\sigma_{j,t}^2 g_t^{i\prime\prime}\left(\sum_{\tau=t+1}^T \gamma_\tau^{i*}\right)}{\zeta_t^i\Omega_t^i},
\end{aligned}$$

where $\Sigma_t^{ii} \equiv \varphi_i^2 + h_i^2\sigma_{i,t}^2 + \sigma_{j,t}^2$, $\Sigma_t^{ij} \equiv h_i\sigma_{i,t}^2 + h_j\sigma_{j,t}^2$, $\Phi_t^i \equiv \frac{k_i^2\left[1+r_i\Sigma_t^{jj}g_t^{i\prime\prime}\right] - r_i\Sigma_t^{ij}y_t^{i\prime\prime}}{k_i^2\left(1-r_i\Sigma_t^{ij}g_t^{i\prime\prime}\right) + r_i\Sigma_t^{jj}y_t^{i\prime\prime}}$, $\Omega_t^i \equiv 1 + r_i\left(\Sigma_t^{ii} + \Phi_t^i\Sigma_t^{ij}\right)g_t^{i\prime\prime}$, and $\zeta_t^i \equiv k_i^2\left(1 - r_i\Sigma_t^{ij}g_t^{i\prime\prime}\right) + r_i\Sigma_t^{jj}y_t^{i\prime\prime}$.

B.2. The stationary model

We examine explicit compensation in a stationary setting where workers' abilities remain unknown to the parties. Fama (1980) argues that the market forces alone can provide adequate incentives to workers to perform, eliminating the moral hazard problems. Assuming $T \to \infty$, Holmström (1999) develops a framework in which in the absence of discounting where workers value equally the current and the "delayed" payments, the stationary effort level is efficient. Suppose that each worker i discounts the future with some factor $\delta \in [0,1]$ and her ability θ^i is drawn from a normal distribution with mean m_1^i. Each worker i's ability fluctuates over the worker's employment and progresses, following the process

$$\theta_{t+1}^i = \theta_t^i + \eta_t^i, \forall i$$

where η_t^i is independent and normally distributed with zero mean and variance σ_η^2. The shocks η_t^i and η_t^j keep adding uncertainty, preventing the market from fully anticipating the level of workers' abilities. Thus, at period $t+1$, worker i's project output is

$$z_{t+1}^i = \theta_t^i + \eta_t^i + e_{t+1}^i + h_j \left(\theta_t^j + \eta_t^j + a_{t+1}^j \right) + \varepsilon_{t+1}^i.$$

The variance of θ_{t+1}^i in this stationary setting, where $\widetilde{\sigma}_{i,t}^2 \equiv \sigma_{i,t}^2 \left(1 - \rho_t^{ii} - h_i \rho_t^{ij}\right)$ is the variance of θ_{t+1}^i before observing the realizations of z_{t+1}^i and z_{t+1}^j. After observing z_{t+1}^i and z_{t+1}^j, the variance along the path to the stationary state of worker i's ability at $t+1$ is $\overline{\sigma}_{i,t+1}^2 = \left(\widetilde{\sigma}_{i,t}^2 + \sigma_\eta^2\right) \widetilde{\mu}_t^i$, where $\widetilde{\mu}_t^i = 1 - \widetilde{\rho}_t^{ii} - h_i \widetilde{\rho}_t^{ij}$. The learning process becomes

$$m_{t+1}^i = \widehat{\mu}_t^i m_t^i + \widehat{\rho}_t^{ii} \left(z_t^i - \widehat{e}_t^i - h_j \widehat{a}_t^j - h_j m_t^j \right) + \widehat{\rho}_t^{ij} \left(z_t^j - \widehat{e}_t^j - m_t^j - h_i \widehat{a}_t^i \right).$$

The correlation coefficients are

$$\begin{aligned} \widetilde{\rho}_t^{ii} &\equiv \frac{\widetilde{\sigma}_{i,t}^2 + \sigma_\eta^2}{\widetilde{\lambda}_t^i} \left[\varphi_i^2 + (1 - h_i h_j) \left(\widetilde{\sigma}_{j,t}^2 + \sigma_\eta^2 \right) \right], \\ \widetilde{\rho}_t^{ij} &\equiv \frac{\widetilde{\sigma}_{i,t}^2 + \sigma_\eta^2}{\widetilde{\lambda}_t^i} \left[h_i \varphi_j^2 - (1 - h_i h_j) h_j \left(\widetilde{\sigma}_{j,t}^2 + \sigma_\eta^2 \right) \right], \end{aligned}$$

where $\widetilde{\lambda}_t^i \equiv \varphi_i^2 \varphi_j^2 + (1 - h_i h_j)^2 \left(\widetilde{\sigma}_{i,t}^2 + \sigma_\eta^2 \right) \left(\widetilde{\sigma}_{j,t}^2 + \sigma_\eta^2 \right) + \varphi_i^2 \left(1 + h_i^2\right) \left(\widetilde{\sigma}_{i,t}^2 + \sigma_\eta^2 \right) + \varphi_j^2 \left(1 + h_j^2\right) \left(\widetilde{\sigma}_{j,t}^2 + \sigma_\eta^2 \right)$. The reputation incentives that arise due to work effort become

$$\begin{aligned} \widetilde{\Xi}_1^{ii} \equiv & \left(1 + h_i - \widetilde{\beta}_1^i - h_i \widetilde{\gamma}_1^i\right) \left(\delta \widetilde{\rho}_1^{ii} + \delta^2 \widetilde{\rho}_1^{ii} \widetilde{\mu}_2^i + \delta^3 \widetilde{\rho}_1^{ii} \widetilde{\mu}_2^i \widetilde{\mu}_3^i + \ldots \right) \\ & - \left(h_j \widetilde{\beta}_1^i + \widetilde{\gamma}_1^i \right) \left(\delta \widetilde{\rho}_1^{ji} + \delta^2 \widetilde{\rho}_1^{ji} \widetilde{\mu}_2^i + \delta^3 \widetilde{\rho}_1^{ji} \widetilde{\mu}_2^i \widetilde{\mu}_3^i + \ldots \right). \end{aligned} \tag{3}$$

In the stationary case where $\widetilde{\rho}_{t+1}^{ii} = \widetilde{\rho}_t^{ii} = \widetilde{\rho}^{ii*}$ and $\widetilde{\rho}_{t+1}^{ij} = \widetilde{\rho}_t^{ij} = \widetilde{\rho}^{ij*}$, equation (3) gives

$$\widetilde{\Xi}_{ii}^* \equiv \frac{\delta}{1 - \delta \widetilde{\mu}_*^j} \left[\left(1 + h_i - \widetilde{\beta}_*^i - h_i \widetilde{\gamma}_*^i\right) \widetilde{\rho}_*^{ii} - \left(h_j \widetilde{\beta}_*^i + \widetilde{\gamma}_*^i \right) \widetilde{\rho}_*^{ji} \right].$$

Similarly, the stationary implicit incentives that arise due to help effort are

$$\widetilde{\Xi}_{ij}^{*} \equiv \frac{\delta}{1-\delta\widetilde{\mu}_{*}^{j}}\left[\left(1+h_i-\widetilde{\beta}_{*}^{i}-h_i\widetilde{\gamma}_{*}^{i}\right)\widetilde{\rho}_{*}^{ij}-\left(h_j\widetilde{\beta}_{*}^{i}+\widetilde{\gamma}_{*}^{i}\right)\widetilde{\rho}_{*}^{jj}\right].$$

Thus, the stationary efforts $\widetilde{e}_{*}^{i}$ and $\widetilde{a}_{*}^{i}$ satisfy the equations

$$g'\left(\widetilde{e}_{*}^{i}\right)=\widetilde{\beta}_{*}^{i}+\widetilde{\Xi}_{ii}^{*} \text{ and } y'\left(\widetilde{a}_{*}^{i}\right)=\widetilde{\gamma}_{*}^{i}+\widetilde{\Xi}_{ij}^{*}.$$

We solve the manager's constraint maximization problem and derive that the stationary contractual parameters satisfy the conditions

$$\left(\frac{\widetilde{\beta}_{*}^{i}}{1-\delta}+\widetilde{\Xi}_{ii}^{*}\right)\left(\sigma_i^2+h_j^2\sigma_j^2\right)+\left(\frac{\widetilde{\gamma}_{*}^{i}}{1-\delta}+\widetilde{\Xi}_{ij}^{*}\right)\Sigma^{ij}+\left(\widetilde{\beta}_{*}^{i}+\widetilde{\Xi}_{ii}^{*}\right)\varphi_i^2=\frac{\delta-\widetilde{\beta}_{*}^{i}-\widetilde{\Xi}_{ii}^{*}}{r\widetilde{g}^{i\prime\prime}}, \tag{4}$$

$$\left(\frac{\widetilde{\beta}_{*}^{i}}{1-\delta}+\widetilde{\Xi}_{ii}^{*}\right)\Sigma^{ij}+\left(\frac{\widetilde{\gamma}_{*}^{i}}{1-\delta}+\widetilde{\Xi}_{ij}^{*}\right)\left(h_i^2\sigma_i^2+\sigma_j^2\right)+\left(\widetilde{\gamma}_{*}^{i}+\widetilde{\Xi}_{ij}^{*}\right)\varphi_i^2=\delta k_i\frac{1-\widetilde{\gamma}_{*}^{i}-\widetilde{\Xi}_{ij}^{*}}{r\widetilde{y}^{i\prime\prime}}. \tag{5}$$

We can multiply both sides of equations (4) and (5) by $1-\delta$. Then, for $\delta=1$ and $r>0$, we derive that the stationary explicit incentives are zero: $\widetilde{\beta}_{*}^{i}=\widetilde{\gamma}_{*}^{i}=0$ for all h_i, h_j, k_i and k_j. In the absence of discounting where the current and the future payments weight equality on a worker's utility, the stationary efforts are driven exclusively by the reputation incentives and are efficient. Thus, Fama's conclusion that the market induces the "right" level of efforts holds, even in our setting in which a worker's career concerns are influenced by the market perceptions about her colleague's ability.

References

Fama, E. F. (1980), 'Agency problems and the theory of the firm', *Journal of Political Economy* **88**(2), 288–307.

Holmström, B. (1999), 'Managerial incentive problems: A dynamic perspective', *Review of Economic Studies* **66**, 169–182.